| Centre Number | | | | | | Candidate Number | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Surname | | | | | | | | | | |
| Other Names | | | | | | | | | | |
| Candidate Signature | | | | | | | | | | |

| For Examiner's Use | |
|---|---|
| Examiner's Initials | |
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| TOTAL | |

General Certificate of Education
Advanced Level Examination
January 2011

# Mathematics MD02

## Unit Decision 2

## Wednesday 26 January 2011 1.30 pm to 3.00 pm

**For this paper you must have:**
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**
- 1 hour 30 minutes

**Instructions**
- Use black ink or black ball-point pen. Pencil or coloured pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Do not write outside the box

Answer **all** questions in the spaces provided.

**1** A group of workers is involved in a decorating project. The table shows the activities involved. Each worker can perform any of the given activities.

| **Activity** | *A* | *B* | *C* | *D* | *E* | *F* | *G* | *H* | *I* | *J* | *K* | *L* |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Duration (days)** | 2 | 5 | 6 | 7 | 9 | 4 | 3 | 2 | 3 | 2 | 3 | 1 |
| **Number of workers required** | 6 | 3 | 5 | 2 | 5 | 2 | 4 | 4 | 5 | 3 | 2 | 4 |

The activity network for the project is given in **Figure 1** below.

**(a)** Find the earliest start time and the latest finish time for each activity, inserting their values on **Figure 1**. *(4 marks)*

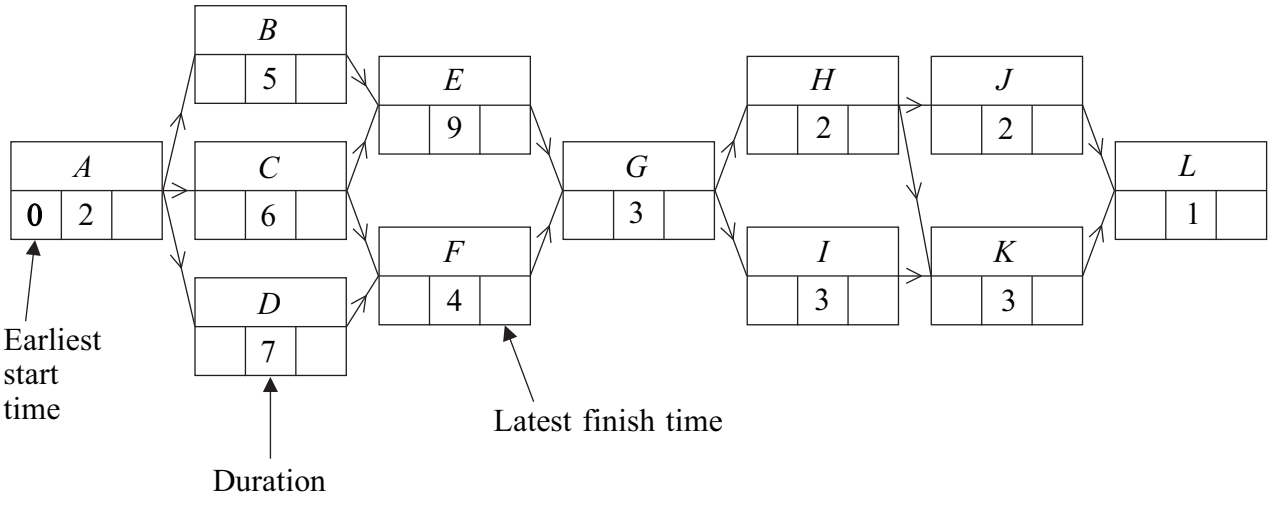
**(b)** Hence find:

**(i)** the critical path;

**(ii)** the float time for activity *D*. *(3 marks)*

**(a)**

**Figure 1**




**(b) (i)** The critical path is .....

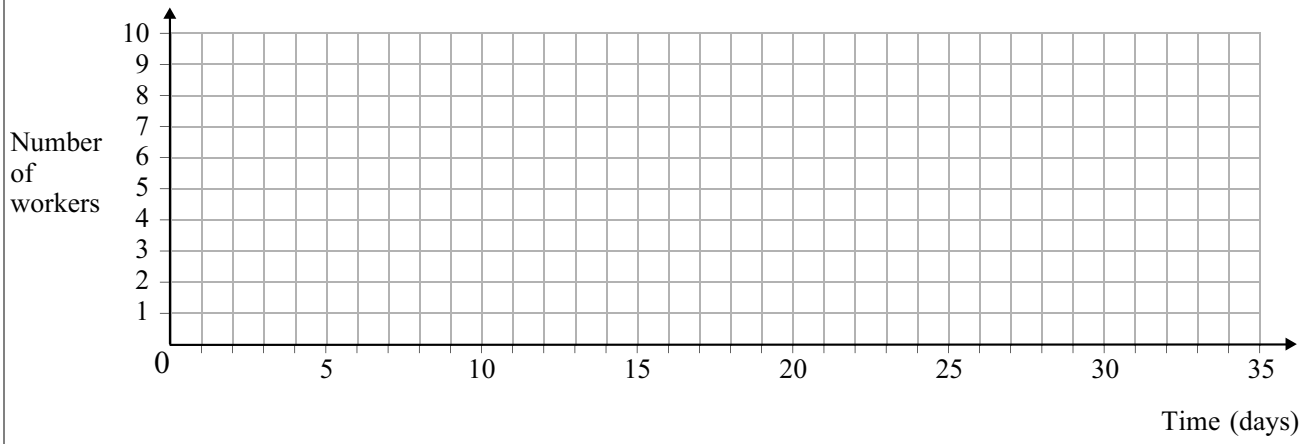
**(ii)** The float time for activity *D* is .....

Do not write outside the box

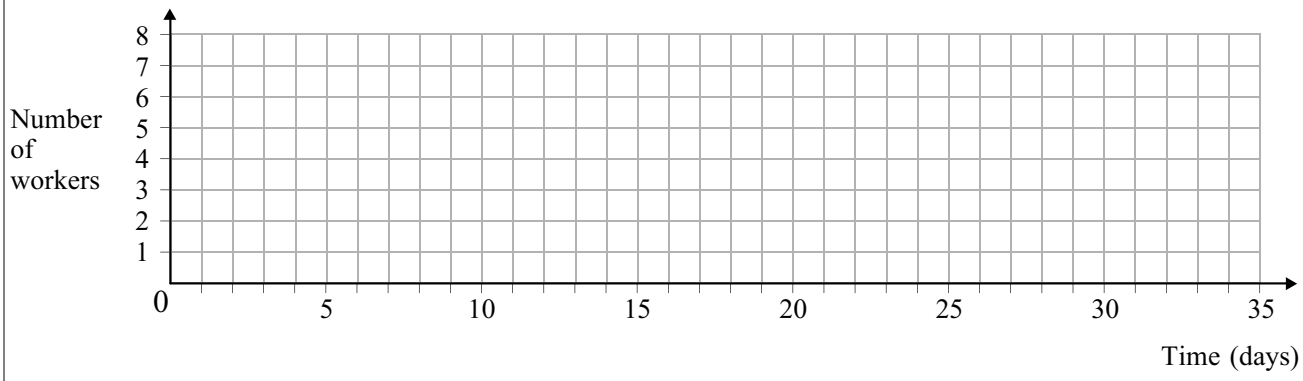
**(c)** Given that each activity starts as early as possible and assuming that there is no limit to the number of workers available, draw a resource histogram for the project on **Figure 2** below, indicating clearly which activities are taking place at any given time. *(4 marks)*

**(d)** It is later discovered that there are only 8 workers available at any time. Use resource levelling to construct a new resource histogram on **Figure 3** below, showing how the project can be completed with the minimum extra time. State the minimum extra time required. *(3 marks)*

**(c)** **Figure 2**




**(d)** **Figure 3**




The minimum extra time required is .....

Do not write outside the box

**2** A farmer has five fields. He intends to grow a different crop in each of four fields and to leave one of the fields unused. The farmer tests the soil in each field and calculates a score for growing each of the four crops. The scores are given in the table below.

| | **Field A** | **Field B** | **Field C** | **Field D** | **Field E** |
|---|---|---|---|---|---|
| **Crop 1** | 16 | 12 | 8 | 18 | 14 |
| **Crop 2** | 20 | 15 | 8 | 16 | 12 |
| **Crop 3** | 9 | 10 | 12 | 17 | 12 |
| **Crop 4** | 18 | 11 | 17 | 15 | 19 |

The farmer's aim is to maximise the total score for the four crops.

**(a) (i)** Modify the table of values by first subtracting each value in the table above from 20 and then adding an extra row of equal values. *(1 mark)*

**(ii)** Explain why the Hungarian algorithm can now be applied to the new table of values to maximise the total score for the four crops. *(3 marks)*

**(b) (i)** By reducing **rows** first, show that the table from part **(a)(i)** becomes

| | | | | |
|---|---|---|---|---|
| 2 | 6 | 10 | 0 | $p$ |
| 0 | 5 | 12 | 4 | 8 |
| 8 | 7 | 5 | 0 | $q$ |
| 1 | 8 | 2 | 4 | 0 |
| 0 | 0 | 0 | 0 | 0 |

State the values of the constants $p$ and $q$. *(2 marks)*

**(ii)** Show that the zeros in the table from part **(b)(i)** can be covered by one horizontal and three vertical lines, and use the Hungarian algorithm to decide how the four crops should be allocated to the fields. *(6 marks)*

**(iii)** Hence find the maximum possible total score for the four crops. *(1 mark)*

QUESTION PART REFERENCE

.....

.....

.....

.....

.....

*Do not write outside the box*

*QUESTION PART REFERENCE*

*Do not write outside the box*

*QUESTION PART REFERENCE*

*Do not write outside the box*

*QUESTION PART REFERENCE*

Do not write outside the box

**3** Two people, Rhona and Colleen, play a zero-sum game. The game is represented by the following pay-off matrix for Rhona.

| | | Colleen | | |
|---|---|---|---|---|
| | ***Strategy*** | $\mathbf{C_1}$ | $\mathbf{C_2}$ | $\mathbf{C_3}$ |
| | $\mathbf{R_1}$ | 2 | 6 | 4 |
| Rhona | $\mathbf{R_2}$ | 3 | $-3$ | $-1$ |
| | $\mathbf{R_3}$ | $x$ | $x+3$ | 3 |

It is given that $x < 2$.

**(a) (i)** Write down the three row minima. *(1 mark)*

**(ii)** Show that there is no stable solution. *(3 marks)*

**(b)** Explain why Rhona should never play strategy $R_3$. *(1 mark)*

**(c) (i)** Find the optimal mixed strategy for Rhona. *(7 marks)*

**(ii)** Find the value of the game. *(1 mark)*

QUESTION PART REFERENCE

*Do not write outside the box*

*QUESTION PART REFERENCE*

Do not write outside the box

**4** The Simplex method is to be used to maximise $P = 3x + 2y + z$ subject to the constraints

$$-x + y + z \leqslant 4$$

$$2x + y + 4z \leqslant 10$$

$$4x + 2y + 3z \leqslant 21$$

The initial Simplex tableau is given below.

| ***P*** | ***x*** | ***y*** | ***z*** | ***s*** | ***t*** | ***u*** | ***value*** |
|---|---|---|---|---|---|---|---|
| 1 | −3 | −2 | −1 | 0 | 0 | 0 | 0 |
| 0 | −1 | 1 | 1 | 1 | 0 | 0 | 4 |
| 0 | 2 | 1 | 4 | 0 | 1 | 0 | 10 |
| 0 | 4 | 2 | 3 | 0 | 0 | 1 | 21 |

**(a) (i)** The first pivot is to be chosen from the $x$-column. Identify the pivot and explain why this particular value is chosen. *(2 marks)*

**(ii)** Perform one iteration of the Simplex method and explain how you know that the optimal value has not been reached. *(5 marks)*

**(b) (i)** Perform one further iteration. *(4 marks)*

**(ii)** Interpret the final tableau and write down the initial inequality that still has slack. *(4 marks)*

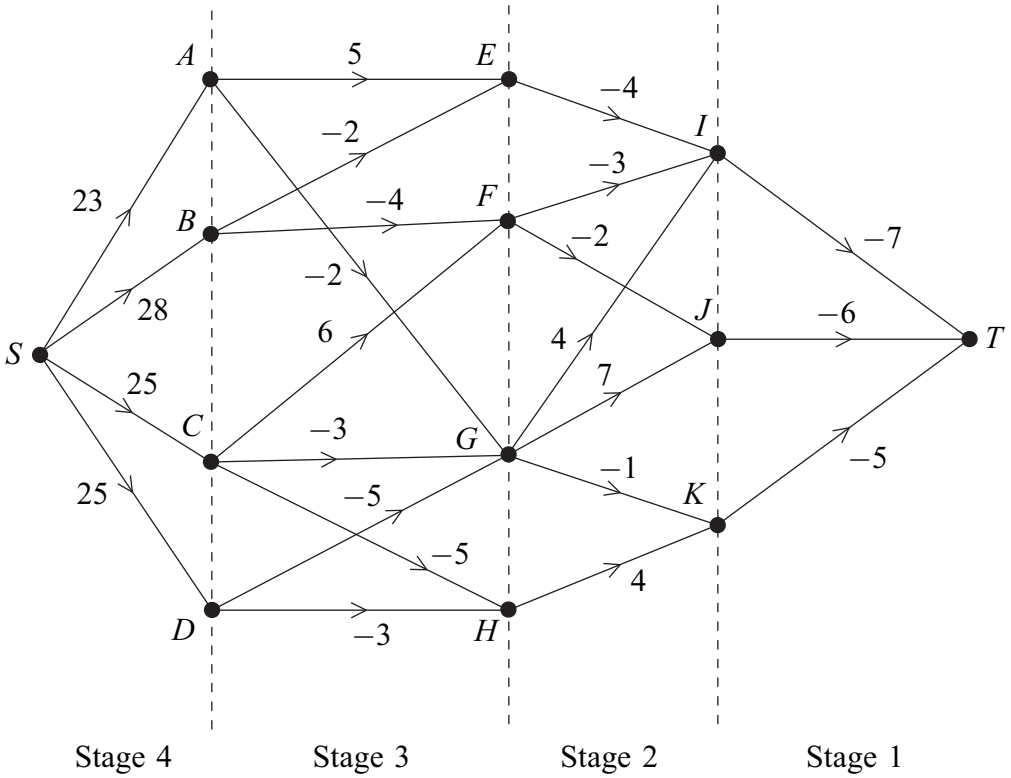
QUESTION PART REFERENCE

*Do not write outside the box*

*QUESTION PART REFERENCE*

Do not write outside the box

**5** Each path from $S$ to $T$ in the network below represents a possible way of using the internet to buy a ticket for a particular event. The number on each edge represents a charge, in pounds, with a negative value representing a discount. For example, the path $SAEIT$ represents a ticket costing $23 + 5 - 4 - 7 = 17$ pounds.




**(a)** **By working backwards from *T*** and completing the table on **Figure 4**, use dynamic programming to find the minimum weight of all paths from $S$ to $T$. *(6 marks)*

**(b)** State the minimum cost of a ticket for the event and the paths corresponding to this minimum cost. *(3 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

**(a)**

**Figure 4**

| Stage | State | From | Value | |
|---|---|---|---|---|
| 1 | $I$ | $T$ | $-7$ | |
| | $J$ | $T$ | $-6$ | |
| | $K$ | $T$ | $-5$ | |
| | | | | |
| 2 | $E$ | $I$ | $-7-4=-11$ | |
| | $F$ | $I$ | | |
| | | $J$ | | |
| | $G$ | $I$ | | |
| | | $J$ | | |
| | | $K$ | | |
| | $H$ | $K$ | | |
| | | | | |
| 3 | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

.....

.....

.....

.....

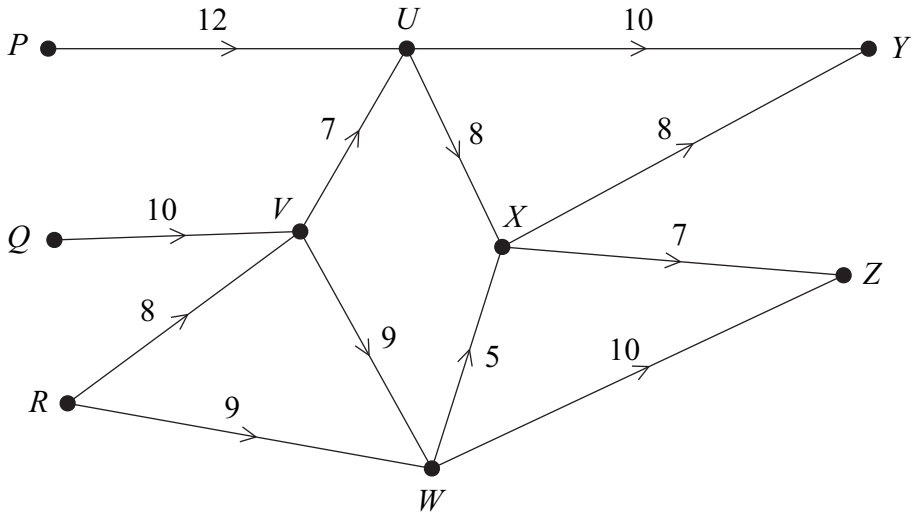
.....

.....

Do not write outside the box

**6** A retail company has warehouses at $P$, $Q$ and $R$, and goods are to be transported to retail outlets at $Y$ and $Z$. There are also retaining depots at $U$, $V$, $W$ and $X$.

The possible routes with the capacities along each edge, in van loads per week, are shown in the following diagram.




**(a)** On **Figure 5 opposite**, add a super-source, $S$, and a super-sink, $T$, and appropriate edges so as to produce a directed network with a single source and a single sink. Indicate the capacity of each edge that you have added. *(2 marks)*

**(b)** On **Figure 6**, write down the maximum flows along the routes *SPUYT* and *SRVWZT*. *(2 marks)*

**(c) (i)** On **Figure 7**, add the vertices $S$ and $T$ and the edges connecting $S$ and $T$ to the network. Using the maximum flows along the routes *SPUYT* and *SRVWZT* found in part **(b)** as the initial flow, indicate the potential increases and decreases of the flow on each edge of these routes. *(2 marks)*

**(ii)** Use flow augmentation to find the maximum flow from $S$ to $T$. You should indicate any flow-augmenting routes on **Figure 6** and modify the potential increases and decreases of the flow on **Figure 7**. *(4 marks)*

**(d)** Find a cut with value equal to the maximum flow. *(1 mark)*

QUESTION PART REFERENCE

.....

.....

.....

.....

.....

Do not write outside the box

**Figure 5**

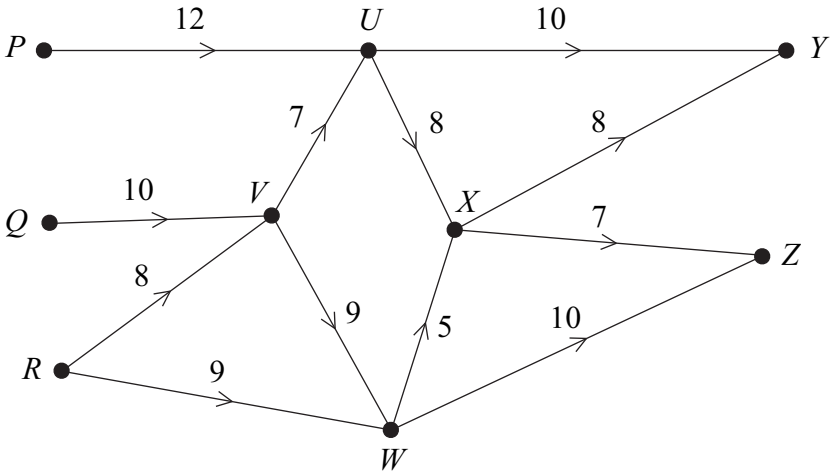
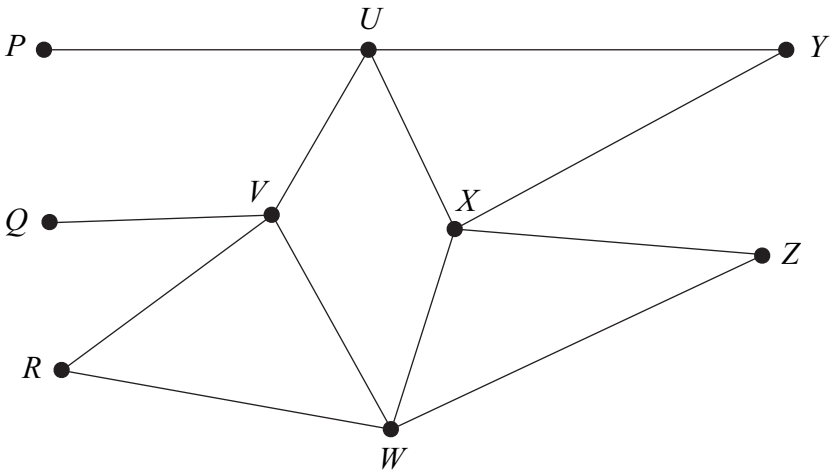



**Figure 6**

| Route | Flow |
|---|---|
| *SPUYT* | |
| *SRVWZT* | |
| | |
| | |
| | |
| | |
| | |
| | |
| | |

**Figure 7**



Do not write outside the box

QUESTION PART REFERENCE

**END OF QUESTIONS**

Copyright © 2011 AQA and its licensors. All rights reserved.